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12. On the volatility of foreign exchange rates[*]

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Summary. We study the interrelationship among three observed phenomena: (i) high volatility of foreign exchange rates, (ii) the large equity premium, (iii) the observed anomaly in the behavior of uncovered interest parity which is known as the "forward discount bias". Using a two country OLG model with a stock, a real bond and two currencies we introduce two exogenous shocks: random net output growth in the home economy and a non-capital endowment growth shock in the foreign economy. Monetary policies are neutral: the domestic policy adjusts the money growth to shocks in domestic output growth while foreign monetary policy adjusts for the foreign endowment shocks. The stock and bond are used as stores of value while money is used only for transactions within each period. Simulations results of Rational Belief Equilibria (RBE) and Rational Expectations Equilibria (REE) show that: (1) Fluctuations of the foreign exchange rate are largely associated with variability in portfolio choices and with flows of financial assets. (2) Endogenous uncertainty is a major cause of the volatility of the exchange rate. This internally propagated uncertainty cannot be explained either by the exogenous variables or by monetary shocks in the two economies. (3) Increased exogenous endowment shocks in the foreign economy have a spillover effect on the financial markets of the domestic economy. In REE this effect *increases* the volatility of commodity and asset prices but in RBE this effect depends upon the beliefs of the agents. An example shows that such spillover effects may *reduce* price volatility in RBE revealing an important feature of RBE. (4) High volatility of the foreign exchange rate in an RBE by itself does not imply a high equity premium or the presence of a forward discount bias. (5) We exhibit a family of RBEs with correlation among the beliefs of agents and a price effect on the probabilities used in different states. For each RBE in this family the foreign exchange rate is volatile, the equity premium is high and the forward discount bias is present. We conclude that the three basic phenomena under examination are all explainable within the RBE paradigm.

JEL Classification Numbers: D58, D84, F31.

1 Introduction

It has been recognized for a long time that foreign exchange rates are more volatile than can be explained by fundamental causes or monetary policy.

Many diverse explanations for this observation have been proposed and we cannot review them all here. A widely quoted explanation is the Dornbusch [1976] model which generates an "overshooting" phenomenon of foreign exchange rates as a result of a slow rate of adjustment of commodity prices to changes in money supply. We shall note in Section 5 below some of the empirical and theoretical arguments against Dornbusch's [1976] price rigidity explanation. An alternative approach associates the fluctuations of foreign exchange rates with the structure of expectations. This literature does not offer any particular theory of expectations but relies on the extensive empirical evidence for the presence of diverse expectations in foreign exchange markets. (See Frankel and Froot [1987], [1990], Frankel and Rose [1995], Froot and Frankel [1989] and the survey by Taylor [1995]; Takagi [1991] reviews the empirical evidence for the presence of diverse expectations in foreign exchange markets). Although we are in general agreement with the second view, this paper goes one crucial step further and proposes the theory of Rational Beliefs due to Kurz [1994] as the specific theory of expectations to be used.

Given the expectational perspective which we propose, the paper studies the dynamics of foreign exchange rates in a Rational Belief Equilibrium (in short, RBE). It shows that, as in a Rational Expectations Equilibrium (in short, REE), exogenous shocks cause some fluctuations in the foreign exchange rate but this effect is very small and cannot explain the observed high volatility of foreign exchange rates. More importantly, the paper shows that variations of the states of beliefs of the agents explain in a natural way the very high volatility of these rates. Moreover, there are two related observed phenomena which have been viewed as puzzles. The first is the high equity premium on asset returns (see Mehra and Prescott [1985]). The second is the anomaly in the market behavior of uncovered interest parity which has also been called the "Forward Discount Bias" in which the regression of the percentage nominal interest differential on the actual percentage change of the exchange rate has a coefficient less than 1. Indeed, Froot [1990] reports that the average estimated coefficient in 75 published papers is -.88 (for a review, see Froot and Thaler [1990] and Engel [1996]). We show in this paper that the RBE theory provides a unified paradigm within which all three observed phenomena are explainable in a natural way.

Our method of analysis is the simulation of general equilibrium patterns of dynamic volatility over time under different parametric specifications. The two country model used in these simulations calibrates the domestic economy to the U.S. parameters. However, the foreign economy is hypothetical in nature and consequently our empirical results are not intended to replicate closely the observed U.S. data or the fluctuations of the exchange rate between the U.S. dollar and any particular currency. For this reason the present paper should be viewed as an exploratory effort with less stringent objectives than those set by Kurz and Beltratti [1997]. Our aim is to study the nature of volatility of foreign exchange rates in an RBE environment and to demonstrate that the model simulations generate results which *are qualitatively*

comparable to the known fact of high volatility of foreign exchange rates, the historically observed equity premium and riskless rate around 5% and 1%, respectively, and a coefficient measuring the forward discount bias which is substantially different from 1.

Studying the behavior of foreign exchange rates in a general equilibrium context requires a formal model of exchange rate determination and this raises an entire spectrum of questions on why multiple currencies should exist to begin with. The Dornbusch [1976] model avoids these questions since it is not derived from individual optimization. Instead, it assumes aggregate demands for goods and multiple currencies, and stipulates capital market equilibrium relationships that must prevail. Negative conclusions regarding the determinacy of exchange rates were reported by Kareken and Wallace [1981] who show that in a symmetric overlapping generations (in short, OLG) model in which agents have a demand for money as a store of value, exchange rates are indeterminate. This conclusion is analogous to the currency substitution conclusions of Boyer [1978] and Girton and Roper [1981]. Manuelli and Peck [1990] extend the negative results to a stochastic OLG economy and Nielsen [1996] shows that the same results hold under rational beliefs. Models with multiple exploding bubble equilibria in foreign exchange markets are reviewed in a survey by Singleton [1987] and more recently studied by King, Wallace and Weber [1992] who examine multiple equilibria in a sunspot economy.

The determination of exchange rates in any model depends crucially upon two components of the model. The first component is the motive to hold money or the technical restrictions imposed by the need to use money in carrying out transactions. The second component is the desire to trade which arises from the *intrinsic difference* between the countries. Starting with the first component we note that most recent research on exchange rate determination in a general equilibrium context is based on Clower's [1967] approach to the demand for money and on its interpretation, in the form of liquidity or cash-in-advance constraints, by Lucas [1980], Stockman [1980] and Helpman [1981]. Papers such as Lucas [1982], Helpman and Razin [1982], [1985] and Svensson [1989] are examples of recent papers which have followed such an approach or variations of it. Obsfeld and Stockman [1987] provides a comprehensive survey of current work on exchange rate dynamics. In our model the demand for money is derived in accord with the above outlined general transaction-liquidity approach. The interpretation of this approach to our particular OLG model is motivated by Lucas [1980].

As for the second component of the exchange rate determination process, there are many differences among countries that could motivate trade. However, differences which explain trade do not necessarily imply a need for *different currencies*. In this paper we study two countries which experience different exogenous shocks and this difference between the stochastic structure of the shocks provides the ultimate reason for the existence of two different currencies. This difference is also related to the structure of monetary policy examined in the model. Our model exhibits growth so that any policy

regarding the money supply has an impact on the equilibrium volatility of the process of prices. If monetary policy aims to be neutral by countering the shocks experienced by the economy, each country must choose a policy which can be responsive *only to one of the shocks* which it faces. The motive for having different currencies is then the result of the desire of each economy to determine a monetary policy that would be responsive to those fluctuations which members of that economy regard as more important to them.

Finally, we have noted that our main tool of analysis here is economic simulation. Due to the fact that this method of theoretical analysis is new, it would be appropriate to clarify at the outset our view on the role of simulations in theoretical work in economics. These comments also apply to the simulation work in the papers of Kurz and Schneider [1996] and Kurz and Beltratti [1997] which are included in this monograph.

Our position is motivated by two basic observations. First, the complexity of the U.S. and the world's economies is so great that one cannot hope to capture all this complexity in simple, single or two agent-type models. Consequently, we are not of the opinion that a simulation of a dynamical model should always aim to *exactly* replicate the real economy. Second, dynamical systems are complex mathematical objects and for this reason it is very difficult to provide a complete characterization of all the relevant quantitative features of a dynamical process using only mathematical tools. Thus, *the objective of all simulation work should be to exhibit the workings of economic principles*. This may entail the investigation of very different questions. On one extreme, a simulation may enable the clarification of the plausible quantitative value of an economic variable when its theoretical properties are reasonably well understood. In such cases we typically want to understand if a variable is likely to take large or small values. On the other extreme, simulation work is a way to understand economic principles through the appearance of *a repetitive regular pattern in diverse models* of simulation. This is simply an alternative way of proving theorems: when examining a new theory one may discover through simulations a regular pattern under very different dynamic conditions and only later exhibit a rigorous mathematical proof that such a pattern would be present under general conditions.

The paper is organized as follows. In Section 2 we present a model of two country trade in which two agent-types are used. In Section 3 we explain the stochastic structure of the model and the construction of the RBE which will be used in the simulations. This section also explains the parametric specifications. In Section 4 we report the simulation results and in Section 5 we discuss some qualifications and suggestions for extending the results.

2 A model of two country trade

2.1 General formulation

We model the trade process by considering two countries: the "home" economy and the "foreign" economy with two different currencies. In each

economy there is a "dynasty" of agent-types which consists, at each date, of one young and one old agent-type. Each agent lives for two periods and therefore there are four agent-types at each date. Although we talk in this paper about "an agent," we use the term agent-type because one must think of the model as postulating a large number of *identical* agents in each country so that no one agent has any effect on equilibrium prices but all within a type have the same endowment, preference and belief. Also, within each dynasty all agents have the same utilities, stochastic processes of endowments and beliefs. This OLG model of dynasties is then similar to the one used in Kurz and Schneider [1996] and Kurz and Beltratti [1997] but with modifications made to accommodate for an international, nominal economy.

The economies have a single, homogeneous consumption good which is freely transferable across the boundary of the two economies. In contrast with Dornbusch [1976], we shall assume that prices are completely flexible and markets clear in equilibrium. We thus show that the Dornbusch [1976] assumption of inflexible prices is not needed to explain the volatility of the foreign exchange rate. The model has two financial assets, the first of which are the ownership shares of an infinitely lived firm. These ownership shares are owned by the public in both economies. The firm itself produces exogenously a stream of net outputs or "dividends" as in Lucas [1978]. The net dividend process $\{D_t, t = 1, 2, \ldots\}$ has the property that

$$D_{t+1} = D_t\, d_{t+1} \tag{1}$$

where $\{d_t, t = 1, 2, \ldots\}$ is a stationary Markov process. The firm is assumed to be located in the home country and the random, exogenous growth rate of dividends characterizes the stochastic structure of the home economy. This has important implications since the dividends are paid in the home currency and consequently dividend payments have an effect on the nominal transactions in the economy. As a result, the random dividend shocks have an impact on market prices. Moreover, since monetary policy in the home country is postulated to be *responsive to domestic shocks*, the dividend process will also have an effect on the evolution of the domestic money supply. We note, however, that ownership shares of the firm will be owned by citizens of both countries and these shares will be traded on an "international" market in both currencies.

Young agents receive an endowment of the homogenous good while old agents receive no endowment. Moreover, the endowment of a young agent born in the home economy at date t, denoted by Ω_t, satisfies the condition

$$\Omega_t = \omega D_t \tag{2}$$

where ω is a constant. This construction is consistent with an interpretation of the growth rate of dividends (or capital income) as being generated by a process of technological innovations. Consequently, the endowment of the young is then interpreted as a net output of all labor and non-capital inputs with rising productivity due to the technological innovations in the economy. The productivity of these non-capital resources then remains in fixed proportion to the random capital output in the home economy. The structure is

different in the foreign economy. In that economy the endowment of a young agent born at date t, denoted by Ω_t^*, satisfies the condition

$$\Omega_t^* = \omega_t^* D_t \tag{3}$$

where $\{\omega_t^*, t = 1, 2, \ldots\}$ is a stationary Markov process which is assumed to be independent of the $\{d_t, t = 1, 2, \ldots\}$ process. Hence, (3) assumes that aggregate endowment of the young in the foreign economy is, *on the average,* proportional to aggregate dividends in the home country. We interpret this to be derived from an implicit assumption that the process of technological innovations in the home economy spreads to the foreign economy and impacts the productivity of resources in the foreign economy.

Assumption (3) says, in addition to the above, that the growth rate of endowment in the foreign economy sustains an exogenous shock which is unique to the foreign economy. This is the mechanism by which we introduce a stochastic structure in the foreign economy which is different from the home country. As a result, monetary policy in the foreign economy is chosen by specifying the shock to which policy makers are required to respond. We shall later assume that changes in the money supply in the foreign economy are responsive to *the endowment shocks experienced in that economy.*

The economic intuition of different exogenous shocks in the two economies is rather compelling. Our model attempts to capture the well documented fact that shocks affect different economies in different ways: some are strongly affected by fluctuations in the yield of crops and natural resources while others are affected by the variability of technology and the productivity of industrial organizations. Since we calibrate the home economy to the U.S. experience, one may interpret the endowment shocks in the foreign economy to be a composite of all other shocks that affect a hypothetical world economy. However, we assume that the diffusion of technological innovations across national boundaries result in the fact that on the average the relative size of the two economies remains the same. We shall stress later that no attempt has been made to calibrate this hypothetical foreign economy to the economy of the "rest of the world."

The second financial asset in the international economy is a zero net supply, short lived bond or "bill" which pays one unit of consumption in the following period. This real debt instrument trades in both currencies and ensures that the model will have a well-defined, riskless rate of interest which, in equilibrium, will be the same in both countries. In the absence of nominal debt instruments in the model, nominal rates are risky rates which depend upon the realized prices in both countries. The main reason for excluding additional nominal debt instruments is computational feasibility. We shall explain this feasibility problem later in Section 5 but observe here that as a consequence of this assumption an RBE has the property that the real system can be calculated independently of the nominal system. This property is common in international finance models (see, for example, Helpman [1981] page 877 and Svensson [1989] page 7) under rational expectations. We observe here that in an RBE with more complex nominal assets, this property may not hold.

We now turn to a preliminary discussion of the demand for money. In the traditional OLG models of a monetary economy (see, for example, Wallace [1980] and Sargent [1987]), money is not being used for liquidity purposes but rather as a store of value used by the young to transfer wealth between two successive points in time. Since our model already contains a common stock and a debt instrument which permit such wealth transfers, in accord with our early discussion we aim to stress the liquidity aspect of the demand for money. However, since in an OLG model new young agents with endowment join in at each date, how do we get an initial money supply into their hands so that they can trade? The approach of cash in advance in its strict form requires the young to acquire cash one period before they need to trade and this is not feasible. The solution which we adopt is inspired by the Lucas [1980] description of the process of transactions and focuses on the demand for money which arises from the process of transactions which need to be executed within each period. Although we describe our solution in detail later, we note that this solution does not require the agents to optimize in the use of cash, but rather it enables the determination of the price level as an equilibrium outcome of the demand for transactions. We can thus turn first to a description of the optimization problems of the agents and only later explain the modeling of the equilibrium price level.

In order to formulate our model we need to explain how we handle the growth aspect in the model. We do that by considering the optimization of the young in the *home country*. Assuming a constant risk aversion and separable utility function, the optimization is stated as follows:

$$\operatorname*{Max}_{(X_1, \theta, B, X_2)} E_{Q_t} \left\{ \frac{1}{1-\gamma} (X_t^1)^{1-\gamma} + \frac{\delta}{1-\gamma} (X_{t+1}^2)^{1-\gamma} | I_t \right\} \tag{4a}$$

subject to

$$p_t X_t^1 + P_t^s \theta_t + q_t^b B_t = p_t \Omega_t \tag{4b}$$

$$p_{t+1} X_{t+1}^2 = \theta_t (P_{t+1}^s + p_{t+1} D_{t+1}) + p_{t+1} B_t. \tag{4c}$$

In (4a)–(4c) X^1 and X^2 denote consumption when young and old, θ is the purchase of ownership shares and B is the number of debt instruments purchased. We stress that no bankruptcy is permitted so that (4a)–(4c) must hold in all states including states on which the agent may place zero probability.

The variables of the system (4a)–(4c) grow without bound and in order for us to use the stability conditions in the sense of Kurz [1994] they need to be normalized. Thus, we divide all equations by D_t and define $x_t^1 = \dfrac{X_t^1}{D_t}$,

$x_t^2 = \dfrac{X_t^2}{D_{t-1}}, b_t = \dfrac{B_t}{D_t}, \omega = \dfrac{\Omega_t}{D_t}, q_t^s = \dfrac{P_t^s}{D_t}$. Using these definitions we rewrite the optimization problem as

$$\operatorname*{Max}_{(x^1, \theta, b, x^2)} E_{Q_t} \left\{ \frac{1}{1-\gamma} (x_t^1)^{1-\gamma} + \frac{\delta}{1-\gamma} (x_{t+1}^2)^{1-\gamma} | I_t \right\} \tag{5a}$$

subject to

$$p_t x_t^1 + q_t^s \theta_t + q_t^b b_t = p_t \omega \tag{5b}$$

$$p_{t+1} x_{t+1}^2 = \theta_t (q_{t+1}^s + p_{t+1}) d_{t+1} + p_{t+1} b_t. \tag{5c}$$

In the rest of this paper we work mostly with normalized variables and we introduce our symmetric notation for the *normalized system*. In the summary below we present pairs of variables where the variables with (*) superscript are associated with the foreign economy.

x_t^1, x_t^{1*} consumption of commodities by the young at t,

x_t^2, x_t^{2*} consumption of commodities by the old at t,

θ_t, θ_t^* stock purchases of the young at t,

b_t, b_t^* bond purchases of the young at t,

ω, ω_t^* commodity endowments of the young at t where ω is constant,

p_t, p_t^* nominal prices of the homogenous commodity at date t,

q_t^s nominal price of the common stocks at date t,

q_t^b price (discounted) of the one period bonds at date t,

$s_t = \dfrac{p_t}{p_t^*}$ exchange rate at t,

I_t, I_t^* information at t.

Having specified this unified notation we can complete the statement of the optimization conditions of the agent in the foreign economy:

$$\underset{(x^{1*}, \theta^*, b^*, x^{2*})}{\text{Max}} E_{Q_t^*} \left\{ \frac{1}{1-\gamma^*} (x_t^{1*})^{1-\gamma^*} + \frac{\delta^*}{1-\gamma^*} (x_{t+1}^{2*})^{1-\gamma^*} | I_t^* \right\} \tag{6a}$$

subject to

$$p_t^* x_t^{1*} + \frac{q_t^s}{s_t} \theta^* + \frac{q_t^b}{s_t} b^* = p_t^* \omega_t^* \tag{6b}$$

$$p_{t+1}^* x_{t+1}^{2*} = \theta_t^* \frac{(q_{t+1}^s + p_{t+1})}{s_{t+1}} d_{t+1} + p_{t+1}^* b_t^*. \tag{6c}$$

Solving (5b)–(5c) for x^1 and x^2 and inserting into (5a), we can write down the first order conditions for the portfolio choice as follows. For the home economy we have

$$\frac{q_t^s}{p_t} (x_t^1)^{-\gamma} = \delta E_Q \left((x_{t+1}^2)^{-\gamma} \left(\frac{q_{t+1}^s}{p_{t+1}} + 1 \right) d_{t+1} | I_t \right) \tag{7a}$$

$$\frac{q_t^b}{p_t} (x_t^1)^{-\gamma} = \delta E_Q ((x_{t+1}^2)^{-\gamma} | I_t). \tag{7b}$$

Similarly, the first order conditions for the foreign economy are

$$\frac{q_t^s}{p_t^* s_t} (x_t^{1*})^{-\gamma^*} = \delta^* E_{Q^*} \left((x_{t+1}^{2*})^{-\gamma^*} \left(\frac{q_{t+1}^s}{p_{t+1}^* s_{t+1}} + 1 \right) d_{t+1} | I_t^* \right) \tag{8a}$$

$$\frac{q_t^b}{p_t^* s_t} (x_t^{1*})^{-\gamma^*} = \delta^* E_{Q^*} ((x_{t+1}^{2*})^{-\gamma^*} | I_t^*). \tag{8b}$$

In equilibrium we must have

$$\theta_t + \theta_t^* = 1 \tag{9a}$$

$$b_t + b_t^* = 0 \tag{9b}$$

$$p_t = s_t p_t^*. \tag{9c}$$

(9a) specifies that the supply of ownership shares is 1 and (9b) specifies that the debt instruments are in zero net supply. (9c) is the arbitrage free pricing establishing purchasing power parity which is applicable to the consumed commodity.

2.2 The firms, the money markets and liquidity constraints

The infinitely lived business firms[1] play a crucial role in the model not only in generating the random dividend process but also in generating the demand for money. The firm generates the dividend process $\{D_t, t = 1, 2, \ldots\}$ and pays, in equilibrium, a cash dividend of $p_t D_t$ to all owners of the shares. We discuss below how the firms get to have the cash to pay the dividend. The firms are also producers of consumer goods which are made for the market. In a more complex model we could assume that the endowments ω and ω_t^* are labor endowments and the firms buy these labor services to produce a consumable commodity. This would enlarge the model and would make computations of general equilibrium more difficult[2]. Since such a productive activity is not central to our model we assume that if a young agent in the home economy consumes x_t^1, he cannot sell the balance of his endowment to other agents but must sell it to the firms. This restriction endows the firms with the technology of turning the amount $(\omega - x_t^1)$ from an endowment of the young into the same number of units of a consumer good that can be exported or sold to the old in the home country. Since there are no explicit production activities in the foreign economy we must imagine firms/central bank in the background which carry out similar activities in the foreign economy. Requiring that commodity flows go through the business firms is an essential component in the transaction structure which generates the demand for money to which we now turn.

Recall first that in our OLG model it is not feasible for the young to satisfy cash in advance constraints since they are "born" with endowment of goods and cannot plan their money requirements. Second, our model already contains common stocks and debt instruments which satisfy the store of value demand for liquid assets. Thus, in order to determine the nominal value of commodities and securities we need to develop a transaction demand for money within each time period rather than across time periods. The problem

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[1] Although in the simulations below there is only one firm we need to think of many such competitive firms operating under free entry. A zero profit condition will be shortly used in the reasoning.

[2] We discuss in Section 5 the problem of model complexity and other compromises which we needed to make in order to be able to carry out the computations.

then becomes: who will hold the supply of money across periods? Our solution in this model is to think that there are central banks in the background which facilitate the transactions in the economy by determining the money supply in each period and providing the liquidity to carry out transactions in each date. At the end of the trading date all money returns to the banks. The demand for money arises from two basic restrictions which we then impose on the flow of transactions in our economy.

Transaction Rules:

(10a) 1. *Every transaction between agents in the economy must be made with money.*

(10b) 2. *All commodities must flow through the firms and agents cannot trade commodities directly. Apart from these restrictions there are no transaction costs.*

To see how these rules work consider the flow of transactions in a typical trading day in the domestic economy part of the model. The foreign operations are exactly symmetric:

(i) at the start of each trading date the central bank makes available to the business firms *free of charge* the money supply M_t. The firms need to carry out two tasks:

(a) pay the young in cash for $(p_t \omega - p_t x_t^1)$, which is the part of their commodity endowment which they want to sell to the domestic firms,

(b) pay out in cash the profits to the owners of the shares.

The objective is to enable all young and old agents to have the liquidity needed to carry out their transactions. Since in an OLG model with a single consumption good the main optimization is carried out by the young, the transactions of the young are the driving forces of our model and we shall explain them in detail.

Transactions by the young. The young use the money received for their endowment to buy the portfolio of securities which is valued, using non-normalized prices, at $P_t^s \theta_t + q_t^b B_t$. We consider first the case $\theta_t \geq 0, B_t \geq 0$. The money spent by the young will flow either to the hands of old agents who own shares or to the hands of the young in the foreign economy who want to borrow (i.e. to sell the debt instruments). Note that all borrowing in the model is international and hence we need to examine the mechanics of international borrowing.

The borrowing foreign agent (who takes a short position in either the stock or the bond) needs to execute a financial obligation which he exchanges for cash in his own country by depositing the obligation into the hands of his own bank. That bank issues foreign currency to the foreign borrower so that if, for example, he is short in bonds then he receives $\dfrac{P_t^s}{s_t}\theta_t^* + \dfrac{q_t^b}{s_t}B_t^*$ units of foreign

currency for the endowment sold to the firms and $-\frac{q_t^b}{s_t}B_t^*$ for the sale of the

negative debt B_t^* to his bank, ending with a total demand for cash of $\frac{P_t^s}{s_t}\theta_t^*$. We make a similar calculation if $\theta_t^* < 0$. In either case, the amount of cash that the foreign agent needs in order to complete the purchase of his portfolio is

$$\text{Max}\left[\frac{P_t^s}{s_t}\theta_t^*, 0\right] + \text{Max}\left[\frac{q_t^b}{s_t}B_t^*, 0\right]. \tag{11}$$

Since at the end of this stage the foreign bank owns the obligation of the young foreign agent, (say, due to a debt instrument when $B_t^* < 0$) the domestic agent who buys the bond will pay the bank domestic currency in exchange for the bond which he receives. The domestic agent who takes a long position in bonds uses for this payment cash which he receives from his domestic firm in exchange for the part $\omega - x_t^1$ of his endowment which he does not consume. The end position in this sequence is that the agents have the liquidity to finance their desired transactions and the domestic cash for the bond is in the hands of the foreign bank. The bank will then use this cash to purchase commodities from the domestic firm and will thus induce an export of these commodities from the domestic to the foreign economy. The foreign bank thus ends up holding commodities as a "backing" to the money it issued to finance the borrowing by the foreign young. These commodities will be sold at the end of the trading day to the owners of the foreign currency who will want to exchange their money for commodities and all the money returns to the bank. It is then clear that borrowing by the foreign agent results in an increase in the demand for money and thus a downward pressure on the price level in the foreign country while at the same time resulting in a demand to export goods from the domestic economy to the foreign economy.

It should now be clear that the case when either $\theta_t < 0$ or $B_t < 0$ is entirely symmetric to the above. In this case the borrowers are the agents in the home country and this borrowing increases the demand for money by the young and the total value of money needed to finance the transactions of the young is

$$\text{Max}[P_t^s\theta_t, 0] + \text{Max}[q_t^b B_t, 0] \tag{12}$$

We then conclude that (11)–(12) sums up the demand for money which is induced by the transactions of the young.

Transactions by the old. We do not carry out an exact accounting of the sequence of transactions carried out by the old. Instead, we set up the accounting of the old so that they have adequate cash to carry out their desired transactions. Now, the old need cash for two purposes: to buy commodities from the business firms and to pay off any debt that they may have. The old receive cash by selling their ownership shares to the young and would have all the cash they need if they receive cash for their dividends. Now, since the use of money is free, competition among the firms ensures zero profits so that the total value of dividends in both economies equals $p_t D_t$. It can now be checked

that once the nominal value of money issued by the central bank is equal to $\text{Max}[P^s_t \theta_t, 0] + \text{Max}[q^b_t B_t, 0] + p_t D_t$ (as in (10a)–(10b) and (11)–(12)), all transactions can be completed: the old have sufficient cash to pay off their debts (merely rearranging the ownership of money among them), and with the final distribution of money they can buy all the consumption goods which they need from the firms. Given the sequence of transactions described here, at the end of the trading day all the money in circulation returns to the firms (and thus to the bank) and all the commodities are consumed by the agents. Next period the money supply is changed by the central bank and the sequence is repeated.

It is true that the total volume of transactions in the home economy exceeds the amount $\text{Max}[P^s_t \theta_t, 0] + \text{Max}[q^b_t B_t, 0] + p_t D_t$ specified. However, the balance of the transactions is counted as "velocity" generated by the purchases of commodities by the old from the business firms and by the cancellation of the debts of the old to each other. Hence, if the nominal money supplies in the two economies are M_t, and M^*_t, then the following conditions must be satisfied in the two economies in equilibrium:

$$M_t = \text{Max}[P^s_t \theta_t, 0] + \text{Max}[q^b_t B_t, 0] + p_t D_t \tag{13a}$$

$$M^*_t = \text{Max}\left[\frac{P^s_t}{s_t} \theta^*_t, 0\right] + \text{Max}\left[\frac{q^b_t}{s_t} B^*_t, 0\right]. \tag{13b}$$

Equations (13a)–(13b) provide us the tool to specify the monetary policies of the two economies in a simple way. Thus, before the opening of trade at each date the central bank can simply issue more money or withdraw money from circulation via the business sector. No agent sustains any direct capital gains or losses and once trading opens a new price level is established.

Given our assumption of full price flexibility we readily accept the proposition that large swings in the money supplies of the two countries will cause fluctuations in the exchange rate. Monetary policies are then selected to be neutral since our objective here is to study those fluctuations of the exchange rate that cannot be explained by changes in the money supplies. For this reason each economy selects a monetary policy which is neutral with respect to the shock that it experiences: no country can accommodate *both* shocks. Hence we study the following policies:

$$\text{Monetary policy of the home economy: } M_t = KD_t \tag{14a}$$

$$\text{Monetary policy of the foreign economy: } M^*_t = K^* \Omega^*_t. \tag{14b}$$

It should be clear that one may study the effect of other policies but this is not the objective of the present paper. Since we use the notation $\dfrac{\Omega^*_t}{D_t} = \omega^*_t$, division of (13a)–(13b) by D_t leads to

$$K = \text{Max}[q^s_t \theta_t, 0] + \text{Max}[q^b_t b_t, 0] + p_t \tag{15a}$$

$$K^* \omega^*_t = \text{Max}\left[\frac{q^s_t}{s_t} \theta^*_t, 0\right] + \text{Max}\left[\frac{q^b_t}{s_t} b^*_t, 0\right]. \tag{15b}$$

Equations (15a)–(15b) complete the specification of the market clearing conditions. We now need to specify the stochastic structure of the economy and the beliefs of the agents.

3 The stochastic structure and the construction of the RBE

The stochastic structure of the model consists both of the process of exogenous shocks to output and endowment $\{(d_t, \omega_t^*), t = 1, 2, \ldots\}$ where $(d_t, \omega_t^*) \in R \times R^*$ and of the process of private signals. For a more detailed explanation see the Editor's *General Perspective*, Kurz and Schneider [1996] and Kurz and Beltratti [1997]. Indeed, except for the endowment shock in the foreign economy, our stochastic structure is similar to the one used in these papers. We denote the pair of private signals of the two agents in the domestic and foreign economies by $(y_t, y_t^*) \in Y \times Y^*$.

Assumption 2.1: The state spaces R, R^*, Y, Y^* are finite and the process $\{(d_t, \omega_t^*, y_t, y_t^*), t = 1, 2, \ldots\}$ is a joint Markov process with a probability Π_{RY} and with marginal processes as follows:

    (i) d_t is a stationary Markov process on R,
    (ii) ω_t^* is a stationary Markov process on R^*
    (iii) y_t is an i.i.d process on Y,
    (iv) y_t^* is an i.i.d. process on Y^*.

As for the beliefs of the agents let the price state space be P so that $(p_t, p_t^*, q_t^s, q_t^b) \in P$. The belief Q is then a probability on $((P \times R \times R^* \times Y)^\infty, \mathscr{B}((P \times R \times R^* \times Y)^\infty))$ and Q^* is a probability on $((P \times R \times R^* \times Y^*)^\infty, \mathscr{B}((P \times R \times R^* \times Y^*)^\infty))$. As explained in Kurz and Schneider [1996], private signals are a tool used to express the non-stationarity of these beliefs. This leads to the following assumption:

Assumption 2.2: Under Q and Q^* the dynamical systems $((P \times R \times R^* \times Y)^\infty, \mathscr{B}((P \times R \times R^* \times Y)^\infty), Q, T)$ and $((P \times R \times R^* \times Y^*)^\infty, \mathscr{B}((P \times R \times R^* \times Y^*)^\infty), Q^*, T)$ are stationary Markov processes.

Under Assumption 2.2 it follows that standard techniques of dynamic programming apply to the optimization of the agents in both economies. The demand functions of agents in the home economy depend upon $(p_t, p_t^*, q_t^s, q_t^b, d_t, \omega_t^*, y_t)$ and the demand functions of the agents in the foreign economy depend upon $(p_t, p_t^*, q_t^s, q_t^b, d_t, \omega_t^*, y_t^*)$. It follows that in this set-up the equilibrium map takes the form

$$\begin{bmatrix} p_t \\ p_t^* \\ q_t^s \\ q_t^b \end{bmatrix} = \Phi^*(d_t, \omega_t^*, y_t, y_t^*) \tag{16}$$

Since the calibration of the economy in the simulation below will be made to conform to that stipulated in Mehra and Prescott [1985], we also specify

Assumption 2.3: $Y = Y^* = \{0, 1\}$, $R = \{d^H, d^L\}$ and $R^* = \{\omega^{H*}, \omega^{L*}\}$. The dividend process and the foreign endowment shocks are independent. The marginal measures of Π_{RY} on $(R^\infty, \mathscr{B}(R^\infty))$ and on $(R^{*\infty}, \mathscr{B}(R^{*\infty}))$ specify these processes to be stationary and ergodic Markov processes with transition matrices

$$\begin{bmatrix} \phi, 1-\phi \\ 1-\phi, \phi \end{bmatrix} \text{ and } \begin{bmatrix} \varphi, 1-\varphi \\ 1-\varphi, \varphi \end{bmatrix} \text{ respectively.}$$

Following Mehra and Prescott [1985], we calibrate the home economy to the U.S. and set $d^H = 1.054$, $d^L = .982$ and $\phi = .43$. Similarly, the marginal measures of Π_{RY} on $(Y^\infty, \mathscr{B}(Y^\infty))$ and on $(Y^{*\infty}, \mathscr{B}(Y^{*\infty}))$ specify these processes to be i.i.d with the probability of $y_t = 1$ being α_1 and the probability of $y_t^* = 1$ being α_2.

The value of φ will be specified and motivated later. Since this parameter specifies a rather simplified view of the exogenous shock in "the rest of the world", we remind the reader of our view of the simulation work: we only aim to gain insight into the nature of the forces which determine economic volatility rather than develop a model that can *exactly* replicate the dynamic behavior of world financial markets.

Given the above, we write down a map Φ identifying the 16 price states of the model. Note that the map below is *not* the equilibrium map Φ^* in (16) which defines the values prices take in equilibrium but rather the listing of what makes up each price state:

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \end{bmatrix} = \Phi \begin{bmatrix} d^H & \omega^{H*} & 1 & 1 \\ d^H & \omega^{H*} & 1 & 0 \\ d^H & \omega^{H*} & 0 & 1 \\ d^H & \omega^{H*} & 0 & 0 \\ d^L & \omega^{H*} & 1 & 1 \\ d^L & \omega^{H*} & 1 & 0 \\ d^L & \omega^{H*} & 0 & 1 \\ d^L & \omega^{H*} & 0 & 0 \\ d^H & \omega^{L*} & 1 & 1 \\ d^H & \omega^{L*} & 1 & 0 \\ d^H & \omega^{L*} & 0 & 1 \\ d^H & \omega^{L*} & 0 & 0 \\ d^L & \omega^{L*} & 1 & 1 \\ d^L & \omega^{L*} & 1 & 0 \\ d^L & \omega^{L*} & 0 & 1 \\ d^L & \omega^{L*} & 0 & 0 \end{bmatrix} \qquad (17)$$

where $d_i = d^H$ for $i = 1, 2, 3, 4, 9, 10, 11, 12$, $d_i = d^L$ for $i = 5, 6, 7, 8, 13, 14, 15, 16$, $\omega_i^* = \omega^{H*}$ for $i = 1, 2, 3, 4, 5, 6, 7, 8$, and $\omega_i^* = \omega^{L*}$ for $i = 9, 10, 11, 12, 13, 14, 15, 16$.

3.1 The stationary measure

We note that under Assumption 2.3 our specification is compatible with the specification in Kurz and Schneider [1996] and Kurz and Beltratti [1997]. We proceed to construct the RBE for the simulation model by specifying the Markov transition matrix representing the stationary measure as follows

$$\Gamma = \begin{bmatrix} \varphi\phi A & \varphi(1-\phi)A & (1-\varphi)\phi A & (1-\varphi)(1-\phi)A \\ \varphi(1-\phi)B & \varphi\phi B & (1-\varphi)(1-\phi)B & (1-\varphi)\phi B \\ (1-\varphi)\phi C & (1-\varphi)(1-\phi)C & \varphi\phi C & \varphi(1-\phi)C \\ (1-\varphi)(1-\phi)D & (1-\varphi)\phi D & \varphi(1-\phi)D & \varphi\phi D \end{bmatrix}$$

(18a)

where (A, B, C, D) are all matrices of the following type:

$$A = \begin{bmatrix} a_1, & \alpha_1 - a_1, & \alpha_2 - a_1, & 1 + a_1 - \alpha_1 - \alpha_2 \\ a_2, & \alpha_1 - a_2, & \alpha_2 - a_2, & 1 + a_2 - \alpha_1 - \alpha_2 \\ a_3, & \alpha_1 - a_3, & \alpha_2 - a_3, & 1 + a_3 - \alpha_1 - \alpha_2 \\ a_4, & \alpha_1 - a_4, & \alpha_2 - a_4, & 1 + a_4 - \alpha_1 - \alpha_2 \end{bmatrix}.$$

(18b)

Kurz and Schneider [1996] and Kurz and Beltratti [1997] explain in detail that the marginals of Γ are indeed as specified in Assumption 2.3. In all that we do here we set $\alpha_1 = \alpha_2 = .5$. It is also clear from (18a)–(18b) that apart from α_1 and α_2, Γ is characterized by 16 parameters but in the simulation work we always assume that $A = C$ and $B = D$. This is the assumption that the process of endowment shocks in the foreign economy is independent of all other random variables.

3.2 Specification of the equilibrium conditions in terms of price states

Given the price state space defined in (17) we can then write the equilibrium conditions in the form used in the computations below. The budget equations in the home economy are then written in price states $i = 1, 2, \ldots, 16$ as

$$p_i x_i^1 + q_i^s \theta_i + q_i^b b_i = p_i \omega \tag{19a}$$

$$p_j x_{ij}^2 = \theta_i(q_j^s + p_j)d_j + p_j b_i. \tag{19b}$$

Similarly, in the foreign economy they now take the form (for $i = 1, 2, .., 16$)

$$p_i^* x_i^{1*} + \frac{q_i^s}{s_i}\theta_i^* + \frac{q_i^b}{s_i}b_i^* = p_i^* \omega_i^* \tag{19c}$$

$$p_j^* x_{ij}^{2*} = \theta_i^* \frac{(q_j^s + p_j)}{s_j}d_j + p_j^* b_i^*. \tag{19d}$$

The first order optimum conditions, the market clearing conditions and the money market clearing equations in price states $i = 1, 2, .., 16$ are then written as

$$\frac{q_i^s}{p_i}(x_i^1)^{-\gamma} = \delta \sum_{j=1}^{16} \left((x_{ij}^2)^{-\gamma} \left(\frac{q_j^s}{p_j} + 1 \right) d_j Q_{ij}^i \right) \tag{19e}$$

$$\frac{q_i^b}{p_i}(x_i^1)^{-\gamma} = \delta \sum_{j=1}^{16} (x_{ij}^2)^{-\gamma} Q_{ij}^i \tag{19f}$$

$$\frac{q_i^s}{p_i^* s_i}(x_i^{1*})^{-\gamma^*} = \delta^* \sum_{j=1}^{16} \left((x_{ij}^{2*})^{-\gamma^*} \left(\frac{q_j^s}{p_j^* s_j} + 1 \right) d_j Q_{ij}^{i*} \right) \tag{19g}$$

$$\frac{q_i^b}{p_i^* s_i}(x_i^{1*})^{-\gamma^*} = \delta^* \sum_{j=1}^{16} (x_{ij}^{2*})^{-\gamma^*} Q_{ij}^{i*} \tag{19h}$$

$$\theta_i + \theta_i^* = 1 \tag{19i}$$

$$b_i + b_i^* = 0 \tag{19j}$$

$$p_i = s_i p_i^* \tag{19k}$$

$$K = \text{Max}[q_i^s \theta_i, 0] + \text{Max}[q_i^b b_i, 0] + p_i \tag{19l}$$

$$K^* \omega_i^* = \text{Max}\left[\frac{q_i^s}{s_i} \theta_i^*, 0 \right] + \text{Max}\left[\frac{q_i^b}{s_i} b_i^*, 0 \right]. \tag{19m}$$

The equilibrium will be completely specified by equations (19a)–(19m) once we specify the probabilities Q^i and Q^{i*} of the two agent-types in (19e)–(19h).

3.3 Rational beliefs

Our development follows again the work of Kurz and Schneider [1996] and Kurz and Beltratti [1997]. By assumption 2.2 the beliefs of the agents are probabilities of joint Markov process on prices and individual private signals. Since the marginals on the private signals are assumed i.i.d. with probabilities α_1 and α_2, it follows that if the two pairs of matrices are (F_1, F_2) for the home agent and (F_1^*, F_2^*) for the foreign agent, then rationality of beliefs requires that

$$\alpha_1 F_1 + (1 - \alpha_1)F_2 = \Gamma, \quad \alpha_2 F_1^* + (1 - \alpha_2)F_2^* = \Gamma. \tag{20}$$

Given this, the following probabilities (where F_1^{ij} is the (i,j) element of F_1)

$$Q_{ij}^i = \begin{cases} F_1^{ij} & \text{if } y_i = 1 \\ F_2^{ij} & \text{if } y_i = 0 \end{cases} \quad Q_{ij}^{i*} = \begin{cases} F_1^{ij*} & \text{if } y_i^* = 1 \\ F_2^{ij*} & \text{if } y_i^* = 0 \end{cases} \tag{21}$$

define Q^i and Q^{i*} in (19e)–(19h); the superscripts in Q^i and Q^{i*} stress the dependence on y_i and y_i^*. We now specify (F_1, F_2, F_1^*, F_2^*) in order to complete the definitions of Q_{ij}^i and Q_{ij}^{i*} in (21). To do that we select (F_1, F_2, F_1^*, F_2^*) by using the 32 parameters $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_{16})$ and $\mu = (\mu_1, \mu_2, \ldots, \mu_{16})$ as follows.

Define the row vectors of A with the notation

$$A^j = (a_j, \alpha_1 - a_j, \alpha_2 - a_j, 1 + a_j - (\alpha_1 + \alpha_2)) \quad j = 1, 2, 3, 4. \tag{22}$$

Using similar notation for B, C and D define the four matrix functions of $z = (z_1, z_2, \ldots, z_{16})$:

$$A_1(z) = \begin{bmatrix} z_1 A^1 \\ z_2 A^2 \\ z_3 A^3 \\ z_4 A^4 \end{bmatrix}, \quad A_2^\phi(z) = \begin{bmatrix} \dfrac{1 - \phi z_1}{1 - \phi} A^1 \\ \dfrac{1 - \phi z_2}{1 - \phi} A^2 \\ \dfrac{1 - \phi z_3}{1 - \phi} A^3 \\ \dfrac{1 - \phi z_4}{1 - \phi} A^4 \end{bmatrix} \tag{23a}$$

$$C_1(z) = \begin{bmatrix} z_9 C^1 \\ z_{10} C^2 \\ z_{11} C^3 \\ z_{12} C^4 \end{bmatrix}, \quad C_2^\phi(z) = \begin{bmatrix} \dfrac{1 - \phi z_9}{1 - \phi} C^1 \\ \dfrac{1 - \phi z_{10}}{1 - \phi} C^2 \\ \dfrac{1 - \phi z_{11}}{1 - \phi} C^3 \\ \dfrac{1 - \phi z_{12}}{1 - \phi} C^4 \end{bmatrix}. \tag{23b}$$

Similarly, define the four matrix functions of $z = (z_1, z_2, \ldots, z_{16})$ as follows:

$$B_1(z) = \begin{bmatrix} z_5 B^1 \\ z_6 B^2 \\ z_7 B^3 \\ z_8 B^4 \end{bmatrix}, \quad B_2^\phi(z) = \begin{bmatrix} \dfrac{1 - (1 - \phi) z_5}{\phi} B^1 \\ \dfrac{1 - (1 - \phi) z_6}{\phi} B^2 \\ \dfrac{1 - (1 - \phi) z_7}{\phi} B^3 \\ \dfrac{1 - (1 - \phi) z_8}{\phi} B^4 \end{bmatrix} \tag{24a}$$

$$D_1(z) = \begin{bmatrix} z_{13} D^1 \\ z_{14} D^2 \\ z_{15} D^3 \\ z_{16} D^4 \end{bmatrix}, \quad D_2^\phi(z) = \begin{bmatrix} \dfrac{1 - (1 - \phi) z_{13}}{\phi} D^1 \\ \dfrac{1 - (1 - \phi) z_{14}}{\phi} D^2 \\ \dfrac{1 - (1 - \phi) z_{15}}{\phi} D^3 \\ \dfrac{1 - (1 - \phi) z_{16}}{\phi} D^4 \end{bmatrix}. \tag{24b}$$

An important restriction is indirectly imposed on all parameter choices by the requirement that *both agents believe that the joint process of all variables is Markov.* Hence, given the definitions (23)–(24) we define the matrices $(F_1(\lambda), F_2(\lambda))$ for $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_{16})$ by

$$
F_1(\lambda) = \begin{bmatrix}
\varphi\phi A_1(\lambda) & \varphi(1-\phi)A_2^\phi(\lambda) & (1-\varphi)\phi A_1(\lambda) & (1-\varphi)(1-\phi)A_2^\phi(\lambda) \\
\varphi(1-\phi)B_1(\lambda) & \varphi\phi B_2^\phi(\lambda) & (1-\varphi)(1-\phi)B_1(\lambda) & (1-\varphi)\phi B_2^\phi(\lambda) \\
(1-\varphi)\phi C_1(\lambda) & (1-\varphi)(1-\phi)C_2^\phi(\lambda) & \varphi\phi C_1(\lambda) & \varphi(1-\phi)C_2^\phi(\lambda) \\
(1-\varphi)(1-\phi)D_1(\lambda) & (1-\varphi)\phi D_2^\phi(\lambda) & \varphi(1-\phi)D_1(\lambda) & \varphi\phi D_2^\phi(\lambda)
\end{bmatrix}
$$

and $F_2(\lambda)$ is then defined by the condition

$$
F_2(\lambda) = \frac{1}{1-\alpha_1}(\Gamma - \alpha_1 F_1(\lambda)).
$$

Using the matrices (23a), (23b), (24a) and (24b) we can finally define the pair $(F_1^*(\mu), F_2^*(\mu))$ for $\mu = (\mu_1, \mu_2, \ldots, \mu_{16})$:

$$
F_1^*(\mu) = \begin{bmatrix}
\varphi\phi A_1(\mu) & \varphi(1-\phi)A_2^\phi(\mu) & (1-\varphi)\phi A_1(\mu) & (1-\varphi)(1-\phi)A_2^\phi(\mu) \\
\varphi(1-\phi)B_1(\mu) & \varphi\phi B_2^\phi(\mu) & (1-\varphi)(1-\phi)B_1(\mu) & (1-\varphi)\phi B_2^\phi(\mu) \\
(1-\varphi)\phi C_1(\mu) & (1-\varphi)(1-\phi)C_2^\phi(\mu) & \varphi\phi C_1(\mu) & \varphi(1-\phi)C_2^\phi(\mu) \\
(1-\varphi)(1-\phi)D_1(\mu) & (1-\varphi)\phi D_2^\phi(\mu) & \varphi(1-\phi)D_1(\mu) & \varphi\phi D_2^\phi(\mu)
\end{bmatrix}
$$

and

$$
F_2^*(\mu) = \frac{1}{1-\alpha_2}(\Gamma - \alpha_2 F_1^*(\mu)).
$$

This concludes the specifications of (21) but we note that in this paper $A^j = C^j$ and $B^j = D^j$.

An important property of the specifications here is the assumption that both agent types believe that $\{\omega_j^*, t = 1, 2, \ldots\}$ is a stationary Markov process which is independent of all other random variables. This assumption is made for a particular reason that will be made clear later. Note that this is not the case with respect to dividends and prices: the private signals of the agents are not independent of prices and dividends and agents do not believe that they are independent: under either of the belief matrices $F_1(\lambda)$ or, $F_1^*(\mu)$, future prices depend upon the private signals of the agents.

3.4 Parameter and other model specifications

We have indicated in Assumption 2.3 that in all our calculations below we set $\phi = .43$, $d^H = 1.054$ and $d^L = .982$ to accord with the specifications in the Mehra and Prescott [1985] model. As a result, the domestic economy in the computations below has similar financial characteristics to the U.S. economy

as in Kurz and Beltratti [1997]. We stress, however, that the foreign economy and the monetary policies in both countries are hypothetical and our objective is the study of dynamic patterns which, we think, are generalizable. The parameterization of the domestic economy is then set just to approximate the order of magnitude of the U.S. financial variables. With this in mind we select the following parametric specifications:

(i) $\alpha_1 = .5$ and $\alpha_2 = .5$. Thus the marginals on the private signals are the same across agents.

(ii) $\omega = 30$. However, ω^{*H} and ω^{*L} are parameters which will be studied below.

(iii) $K = 20$ and $K^* = 1$. However, since the mean value of ω^{*H} and ω^{*L} will be assumed below to be 30, the effective money supply in the foreign economy is 30.

(iv) $\gamma = 3, \gamma^* = 3, \delta = .9$ and $\delta^* = .9$. These introduce a basic symmetry in the demand structure of the model.

(v) $\varphi = .6$. This introduces some persistence of the exogenous endowment shocks in the foreign economy.

(vi) In the model at hand *REE* are defined by requiring the agents to ignore their private signals and set their beliefs by selecting

$$\lambda_i = 1 \text{ and } \mu_i = 1 \quad \text{for } i = 1, 2, \ldots, 16$$

$$a_i = b_i = c_i = d_i = .25 \quad \text{for } i = 1, 2, 3, 4.$$

(vii) *Rational belief equilibria* are studied here by considering two sets of parameters which we now discuss.

A. Intensity parameters. These are identified by λ_i and by μ_i which specify how the agents interpret their private signals. In the calculations below we consider two configurations. The first one is

$$\text{(I): } \lambda_i = \mu_i = 1.75 \quad \text{for } i = 1, 2, \ldots, 16. \tag{25}$$

The interpretation of (25) is very simple: if the private signals at t take the values $y_t = 1$ for the home agent or $y_t^* = 1$ for the foreign agent, the agents increase (respectively) their assessment of the probabilities of $(p_1, p_2, p_3, p_4, p_9, p_{10}, p_{11}, p_{12})$ at $t+1$ by a factor of 1.75 relative to the probabilities of these prices in the stationary measure. Hence, when $y_t = 1$ or $y_t^* = 1$ an agent becomes more optimistic about the occurrence of these 8 price states at $t + 1$ relative to the complementary 8 states. Since these 8 states are also the states at which $d_t = d^H$, one may also think about this configuration as one in which the agents become optimistic about states of high dividends next period. It is central to configuration I that the fluctuations between states of "optimism" and states of "pessimism" occur without using *observed current prices* to interpret the signals since they result only from the values taken by y_t and y_t^*. For this reason we shall refer to this type of an RBE as one in which *there are no price effects* on the change in agents' probabilities (relative to the stationary measure) induced by a state of beliefs.

In configuration II the agent considers the value taken by p_t in the interpretation of his private signals. More specifically, the price effect stipulates that the agent's interpretation is contingent on whether p_t takes any of the values p_1, p_5, p_9 or p_{13} in the following manner:

- Become optimistic about the occurrence of $(p_1, p_2, p_3, p_4, p_9, p_{10}, p_{11}, p_{12})$ at $t + 1$ if $y_t = 1$ (or $y_t^* = 1$), and if, in addition, p_t is *not* equal to p_1, p_5, p_9 or p_{13}.

- Become pessimistic about the occurrence of $(p_1, p_2, p_3, p_4, p_9, p_{10}, p_{11}, p_{12})$ at $t + 1$ if $y_t = 1$ (or $y_t^* = 1$) and if, in addition, p_t is *also* equal to p_1, p_5, p_9 or p_{13}.

The above price dependency of the beliefs is equivalent to the vectors λ and μ taking the form[3]:

$$(\text{II}): \lambda = \mu = (.25, 1.75, 1.75, 1.75, .25, 1.75, 1.75, 1.75, .25, 1.75, 1.75, 1.75, .25, 1.75, 1.75, 1.75).$$

(26)

B. Correlation Parameters. These are the vectors a, b, c and d specified earlier in the matrices A, B, C, and D. As we noted above, in rational expectations equilibria $a = b = c = d$ and each one of these parameters equals .25. Deviations from .25 introduce correlation between y_t and y_t^* and when the matrices A, B, C, and D are not equal to each other, the correlation between these private signals may be dependent upon prices, dividends or endowment shocks in the foreign economy. Apart from the rational expectations case, we shall mostly consider a pattern of correlation determined by the specifications

$$a_1 = .001, \ a_2 = .499, \ a_3 = .499, \ a_4 = .001$$
$$b_1 = .499, \ b_2 = .499, \ b_3 = .499, \ b_4 = .499 \tag{27}$$
$$A = C, \ B = D.$$

(viii) *The forward discount bias coefficient β.* The desired parameter is the theoretical regression coefficient of the differential between the nominal interest rate in the two countries at each date t on the realized percentage currency depreciation between dates t and $t + 1$. Unfortunately, as we shall discuss in Section 5, our model has only a single real interest rate (in commod-

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[3] Note that the specific definition (see (23)–(24)) of the parameters λ and μ restricts us from considering more complex structures in which the interpretation of private signals is contingent on prices. In order to see the range of possible models one can construct, consider the following example. Suppose we want to describe an agent who, when optimistic, extrapolates the occurrence of high prices and suppose that in some equilibrium the set H of prices is the set of "high" prices. We can then define a vector λ^H of parameters such that λ_t^H is the (scalar) proportion by which the agent adjusts the probabilities of next period's prices in the set H (rather than the four prices $\{1, 2, 3, 4\}$ as it is in (23a) above) relative to the stationary measure, given that he observes price i. If he sets $\lambda_t^H > 1$ for $i \in H$ and $\lambda_t^H < 1$ for $i \notin H$ then we interpret this behavior to state that to be optimistic means to assign higher probability to the *persistence* of high prices and to be pessimistic means to assign lower probability to the persistence of high prices.

ity terms) and no nominal debt instruments to determine equilibrium nominal rates. However, by averaging over the price level risks of the bond returns over the 16 states we hope to approximate the riskless nominal return. That is, at date t an agent in the home economy can purchase a riskless real bond for the nominal price of q_t^b. This bond will pay the nominal amount of p_{t+1} at date $t+1$ yielding a risky nominal rate of $\dfrac{p_{t+1} - q_t^b}{q_t^b}$. Based on these considerations we approximate the riskless nominal interest rate at state i in the home and foreign economies by

$$r_i^N = \sum_{j=1}^{16} \frac{p_j - q_i^b}{q_i^b} \Gamma_{ij} \tag{28a}$$

$$r_i^{N*} = \sum_{j=1}^{16} \frac{p_j - \dfrac{q_i^b}{s_i}}{\dfrac{q_i^b}{s_i}} \Gamma_{ij}. \tag{28b}$$

Given these definitions of the nominal rates in the two economies at i we can then compute the regression coefficient β of $(r_i^N - r_i^{N*})$ on $\dfrac{1}{s_i}(s_j - s_i)$ using the transition probability matrix Γ and the implied equilibrium stationary distribution.

4 Simulation results

4.1 REE: the effect of foreign endowment shocks on the domestic financial markets

The first question which we study is the effect of increasing the size of the foreign endowment shocks (i.e. ω_t^*) on the domestic financial markets. The foreign endowment shock is a proxy for all possible fluctuations such as business cycles, the effects of wars, discoveries of natural resources, fluctuations in crop yields, technological shocks etc. This type of spillover effect is then central to the formulation of trade policy and provides the central motive for counter-cyclical foreign trade policy. Since we do not have unemployment in the model, our focus here is on the financial consequences of such spillovers and on the differences between the effects of such spillovers on REE and RBE.

In Table 1 we report the results of increasing the difference $(\omega^{H*} - \omega^{L*})$ of the foreign endowment shock from 0 to 12 when the mean is 30. This range of the shock is extraordinarily large: it aims to illustrate a theoretical point, not to be empirically realistic. Apart from variables that have already been defined, the following variables are also reported in the table:

σ_p^2 –   the long term variance of the domestic commodity price level,

σ_{p*}^2 –   the long term variance of the foreign commodity price level,

$\bar{S}, \bar{p}, \bar{p}^*$ — the mean exchange rate and price levels in home and foreign economies, respectively,

σ_s^2 — the long term variance of the foreign exchange rate,

$\bar{r}^F$ — the mean real riskless rate

ρ — the real equity premium

β — the implied regression coefficient of the nominal interest rate differential at each date on the realized currency depreciation between date t and $t+1$.

Turning to Table 1 we note that the mean values of prices and the exchange rate do not change as the foreign shocks increase in size. This should clarify the fact that the monetary policies of both countries ensure that in equilibrium the economies *do not experience any long term inflation.* Moreover, the mean values of prices and the foreign exchange rate do not change with the size of the shocks since the monetary policy adopted in the foreign economy was constructed so as to adjust the growth of the money supply to the endowment shocks and hence all fluctuations in commodity prices are the result of income effects and portfolio adjustments.

To see the important result exhibited in Table 1 note that the rising aggregate uncertainty in the foreign economy translates into a rising aggregate uncertainty in the international market and the consequences are direct: rising commodity price volatility, rising volatility of the foreign exchange rate, falling

Table 1. Rational expectations equilibrium: The effects of foreign endowment shocks on domestic financial markets and on the foreign exchange rate

ω^{H*}	30	31	32	33	34	35	36
ω^{L*}	30	29	28	27	26	25	24
$\frac{1}{\omega^{L*}}[\omega^{H*}-\omega^{L*}]$	0	6.9%	14.3%	22.2%	30.8%	40%	50%
$\bar{p}$	1.291	1.291	1.291	1.290	1.290	1.288	1.286
σ_p^2	.00001	.00007	.00025	.00055	.00097	.00151	.00217
$\bar{p}^*$	2.071	2.070	2.070	2.068	2.067	2.064	2.062
$\bar{s}$	.624	.624	.624	.624	.624	.624	.624
σ_s^2	.001E-5	.008E-5	.028E-5	.061E-5	.108E-5	.168E-5	.241E-5
$\bar{r}^F$	4.88%	4.77%	4.46%	3.94%	3.22%	2.33%	1.27%
ρ	.45%	.60%	1.05%	1.79%	2.82%	4.12%	5.67%
β	.9535	.9543	.9575	.9611	.9669	.9741	.9828

riskless rates and rising equity premia on risky assets. However, to get the equity premium to a level above 5% we have to push the shocks to the 36–24 range. This range has extraordinary consequences which can be seen as follows. Recall that

$$\frac{\Omega_t}{\Omega_{t-1}} = d_t \quad \text{and} \quad \frac{\Omega_t^*}{\Omega_{t-1}^*} = \frac{\omega_t^*}{\omega_{t-1}^*} d_t.$$

Now, since the fluctuations in d_t (i.e. 1.054 and .982) have been calibrated by Mehra and Prescott [1985] to the real fluctuations of the U.S. economy, the values of 36 and 24 taken by ω_t^* imply that the growth factor in the hypothetical foreign economy takes the four values of 1.581, 1.473, .706 and .658!! This is really an alternative way of encountering the equity premium puzzle: in an REE we need to push the endowment shock in the foreign economy to totally unreasonable levels in order for the model to generate an equity premium over 5%. But note now the analogous conclusion with respect to the fluctuations of the foreign exchange rate. First, as the level of the foreign endowment shock rises, the variance of both the domestic as well as the foreign price levels rise. However, in the REE we need to bring the shock to the high range of 36–24 for the domestic price level to exhibit fluctuations in the minimal range between 1.236 and 1.336 and for the foreign price level to fluctuate between 1.977 and 2.148 (standard deviations of about .046 in the home economy and .081 in the foreign economy). Second, the fluctuations of the foreign exchange rate are negligible at *any range* of the foreign exogenous shocks. Even at the range of 36–26 the variance of the foreign exchange rate is only .00000241.

The important conclusion one must draw from Table 1 is that the equity premium puzzle is a general phenomenon according to which all prices, both assets as well as commodity prices, fluctuate in the real economy of our daily life much more than in models constructed under rational expectations. Although the domestic economy is calibrated to the U.S. economy we find that even if we generate extraordinary shocks in the foreign economy we cannot generate in an REE any reasonable fluctuations of commodity prices or of the foreign exchange rate.

As one would expect, in an REE there is no forward discount bias and β is close to 1 where the difference is due to the risk aversion of the agents. We also note that our analysis here also shows that in a general equilibrium context the foreign sector is an important source of economic volatility which contributes to the equity premium in the domestic economy[4].

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[4] This comment applies to models with heterogenous agents and multiple commodities. The comment does not mean that the analysis of the equity premium with a single household economy and a homogeneous consumption good as in Mehra and Prescott [1985] would have benefitted from the introduction of a foreign sector into the analysis. Such a model takes the fluctuations of consumption as exogenous ignoring the sources of such volatility. This is not possible in a model with heterogeneous commodities and heterogeneous agents.

4.2 RBE I: The effect of foreign endowment shocks on the domestic financial markets

The REE analyzed in Table 1 provide a general background for the study of the volatility characteristics of RBE. Our main aim here is to explore through examples some characteristics of RBE which are important for the understanding of how such equilibria work. We thus start with the following simple structure of rational beliefs:

(i) the intensity parameters are $\lambda_i = \mu_i = 1.75$ for $i = 1, 2, \ldots, 16$ (29a)

(ii) the correlation parameters are $a_i = b_i = c_i = d_i = .25$ for $i = 1, 2, 3, 4$.   (29b)

Under this structure of beliefs prices have no effect on the determination of the individual states of belief as outlined in Section 3.4. The results in Table 2 focus on volatility measures and average price *levels* are omitted since the pattern of variations of these means is not different from the pattern in Table 1.

There are several sharp differences between the results reported in Table 1 and those reported in Table 2. We first state these differences and then discuss them one at a time. First, the level of price volatility in Table 2 is much larger than the level of price volatility in Table 1. For example, comparing the results for the 31–29 column in both tables we see that the variance of commodity prices in the RBE is .227 compared with .00001 in the corresponding REE. However, the variances σ_p^2 and σ_{p*}^2 of prices in the table are unreasonably large relative to the observed fluctuations in the domestic price levels in the U.S. and other major economies. Note also that the difference between σ_p^2 and σ_{p*}^2 has no significance: it only reflects the difference between the means. It is obvious that

Table 2. Rational belief equilibrium (I) without price effects: The effects of foreign endowment shocks on volatility characteristics

ω^{H*}	30	31	32	33	34	35	36
ω^{L*}	30	29	28	27	26	25	24
$\dfrac{1}{\omega^{L*}}[\omega^{H*} - \omega^{L*}]$	0	6.9%	14.3%	22.2%	30.8%	40%	50%
σ_p^2	.227	.224	.210	.149	.093	.059	.042
σ_{p*}^2	.601	.594	.559	.404	.254	.171	.128
σ_s^2	.01384	.01454	.01899	.03871	.08102	.08237	.05501
$\bar{r}_F$	4.95%	4.95%	4.91%	4.35%	3.63%	2.73%	1.68%
ρ	.78%	.74%	.77%	1.45%	2.46%	3.75%	5.30%
β	.376	.384	.409	.563	.755	.827	.838

there is an order of magnitude difference between the variances of the exchange rates in the RBE's of Table 2 and the variances of the foreign exchange rates in the REE's of Table 1. In addition, one should note that the variances of the exchange rate are very large since a variance of, say, .04 in the exchange rate means a standard deviation of .2. For a foreign exchange rate with a mean of around .6 such standard deviation means that it fluctuates mostly between .4 and .8 which is an extremely large range.

The second sharp difference between the results in Table 1 and those in Table 2 is related to the pattern of the impact of the rising size of the foreign endowment shocks on volatility characteristics. In the REE (Table 1) the rising shocks *increase* the volatility of commodity prices. The results for the RBE in Table 2 are the opposite: the effect of the rising foreign shocks is to *reduce* the volatility of commodity prices. In both cases the volatility of the foreign exchange rate does not change monotonically with respect to the variance of the foreign endowment shock: it increases at first but the level of exchange rate volatility peaks when the foreign endowment is around 35–25. The third sharp difference is the appearance of a significant forward discount bias in the last row of Table 2.

It is important to note that the RBE in Table 2 and the REE in Table 1 have basically the same patterns of riskless rates and equity premia. In both sets of equilibria the riskless rate at 30–30 is close to 5% and it falls sharply as the size of the foreign shock increases. Similarly, the equity premium at 30–30 is less than 1% and rises sharply as the size of the shocks increase. We repeat an observation made by Kurz and Beltratti [1997]: an increased equilibrium price volatility by *itself* does not imply a fall in the riskless rate and an increased equity premium.

We comment first on the large price and exchange rate volatility in the RBE's in Table 2. Note that the equilibrium map (16) shows that the large price volatility in the RBE is the result of fluctuations in the state of beliefs of the agents. Indeed, a comparison of Tables 1 and 2 shows with great clarity the main conclusion of this paper which states *that endogenous uncertainty is the dominant form of uncertainty in foreign exchange markets*! To express this conclusion differently, recall that under the assumption of complete price flexibility and neutral monetary policy the standard result under rational expectations holds that fluctuations of the price levels and of the exchange rate would be minor. This is exactly what Table 1 shows. In an RBE the fluctuations of prices and the exchange rate are, in addition, the result of changes in the state of beliefs of the agents. Such changes lead agents to alter their consumption plans and adjust their portfolios. These fluctuations in the structure of demand cause fluctuations both in the relative prices of securities to commodities as well as in the structure of transactions in the economy. These induce changes in the demand for money and consequently result in changes in the price level as well. We stress that the endogenously propagated component of exchange rate volatility in an RBE is generated *in addition* to any fluctuations that may be induced by monetary policy. Finally, we noted that the variances of commodity prices in Table 2 are unreasonably high

relative to the experience in the major economies and we comment on this issue in Section 5.

Why does price volatility in Table 2 fall with the size of the exogenous shocks? To help the reader gain some intuition into the workings of an RBE, we recall the assumption made earlier which stipulated that the endowment shocks were independent of other variables and that the agents believed that such independence was indeed the case. This implies that *the beliefs of the agents with respect to the foreign endowment shocks were the truth*. Now, in any RBE the mechanism which generates volatility is the time variability of the state of beliefs and disagreement among the agents. When we introduce the exogenous variability of the endowment shocks this variability may *interact* with the effects of the states of belief or it may *negate* it. The net effect depends upon what the agents believe about the exogenous process. On one hand, we note that if the agents believe that the process is independent of their own signals and of the dividend process then, as these exogenous shocks get larger, their forecasts of the future become more dominated by this exogenous process. On the other hand, if the agents are in agreement regarding the stochastic nature of this process, then they find that their forecasts about the future are getting more and more similar as the size of this dominant effect grows. This growing similarity of beliefs about availability of future supplies and its convergence to the truth reduces the endogenous effect of states of belief, removes trading opportunities and reduces commodity price volatility. We can think of the increased dominance of the exogenous shocks acts as a *focal point of coordination* within the RBE which reduces the relative effect of endogenous uncertainty.

We consider the focal point effect reported above as one of the important observations of this paper since it suggests a more general principle. That is, it proposes the idea that if within an RBE there exists an independent exogenous random variable with respect to which agents have full structural knowledge then, as the variance of this variable rises, the volatility characteristics of the RBE become more similar to an REE where the effects of endogenous uncertainty vanish relatively. Hence, one may start with a very volatile RBE but as the effect of the focal point variable rises, the endogenous variability gives way to a more coordinated view of the market by all participants. If general, such a conclusion could have important implications which we plan to study in the future.

We make a final comment as an introduction to our next investigation. The configuration of beliefs formulated in (29a)–(29b) has no dependency on prices in the sense explained earlier. In addition, due to (29b) the private signals of the agents are uncorrelated and consequently the fluctuations between states of optimism and states of pessimism may be viewed as pure "social noise" although the agents are entirely rational and use all available market information. As a result there is no equilibrium with a reasonable variance of the foreign endowment process in either Tables 1 or Table 2 which exhibits all three observed phenomena specified at the start of the paper: (i) high volatility of foreign exchange rate, (ii) high equity premium with low riskless rate and (iii)

a value of β significantly below 1 representing the forward discount bias. Thus, from the point of view of the RBE theory, the pattern of beliefs (29a)–(29b) generates a great deal of volatility but does not explain the observed phenomena. The RBE's reported in the next section do exhibit the desired results. They are generated by structures of beliefs with two essential properties: *they have price effects and their private signals are correlated.*

4.3 RBE (II) With price effects: exhibiting all three empirical phenomena

Table 3 presents the results of the effect of the endowment shocks on RBE with a structure of belief as follows: the intensity parameters are of type (II):

$$\lambda = \mu = (.25, 1.75, 1.75, 1.75, .25, 1.75, 1.75, 1.75, .25, 1.75, 1.75, 1.75, .25, 1.75, 1.75, 1.75)$$

and the correlation parameters are as specified in (27):

$$a_1 = .001, a_2 = .499, a_3 = .499, a_4 = .001$$

$$b_1 = .499, b_2 = .499, b_3 = .499, b_4 = .499$$

$$A = C, B = D.$$

If we restrict attention to values of the exogenous shocks which imply a modest range of variations of, say, less than 10% as in the first two columns, then the results in Table 3 show that the RBE's corresponding to these columns satisfy the stipulated conditions. The variance of the foreign exchange rate is large; the risk premium is over 4.6%; the riskless rate is less than 1.82% and the forward discount bias parameter is less than .24.

Table 3. Rational belief equilibrium (II) with price effects: The effects of foreign endowment shocks on volatility characteristics

ω^{H*}	30	31	32	33	34	35	36
ω^{L*}	30	29	28	27	26	25	24
$\dfrac{1}{\omega^{L*}}[\omega^{H*} - \omega^{L*}]$	0	6.9%	14.3%	22.2%	30.8%	40%	50%
σ_p^2	.182	.178	.170	.150	.088	.050	.036
$\sigma_{p^*}^2$	.482	.473	.453	.402	.247	.150	.107
σ_s^2	.00786	.00881	.01166	.01951	.04466	.05751	.05000
$\bar{r}_F$	1.82%	1.80%	1.87%	2.02%	1.46%	.65%	−.35%
ρ	4.81%	4.67%	4.39%	4.03%	4.72%	5.87%	7.35%
β	.2243	.2396	.2620	.3139	.5065	.6401	.6882

It is clear that the forward discount bias is not close to the average estimate of −.88 reported by Froot [1990] and therefore the results reported here should be viewed as a qualitative demonstration that the endogenous uncertainty paradigm is consistent with all three empirical observations under discussion. To support this conclusion we note that Kurz and Beltratti [1997] discuss the mechanism by which beliefs which are price dependent and correlation among private signals generate RBE's with low riskless rates and high premia. Indeed, these appear to be the essential conditions needed for the observed phenomena to be replicated in models with endogenous uncertainty. These arguments remain valid explanations of the results in Table 3. We now want to explain why the theory of endogenous uncertainty can also explain the forward discount bias in foreign exchange markets.

The argument leading to the forward discount bias has two separate components. The first one is the statement that in an equilibrium with a complete market structure the difference between the nominal interest rates in the two countries would be *exactly* equal to the difference between the two forward rates in the two countries. The truth of this proposition arises from a simple arbitrage argument and if it failed to hold in any market situation one would need to examine what limitations on trade caused such arbitrage activity from taking place. The second part of the forward discount bias is derived from the proposition that under rational expectations, the difference between the forward rates would be exactly equal to the expected percentage depreciation of the exchange rate between the two countries. This *expectational* argument then leads to the statistical model that seeks to establish that, apart from compensation for risk aversion, the differential between the one period nominal rates in the two countries would be an unbiased estimate of the one period depreciation of the exchange rate. Under this proposition one would then expect to have a regression coefficient of 1 (apart from risk aversion) between the percentage differential of the nominal rates and the actual percentage change of the exchange rate.

The theory of RBE denies the truth of the second part of this argument. It predicts that, in general, disagreement among the agents would result in a true, equilibrium, process of the exchange rate which would fluctuate due to variations in the state of beliefs. Hence, at any date the nominal interest differential between the two countries will not be an unbiased estimate of the rate of change of the exchange rate and under such circumstances one should not expect the regression coefficient to be close to one. Agents who may want to take advantage of such a regression, basing their investment strategy on a nominal rate differential which appears to offer an arbitrage opportunity, will find that this is not arbitrage in the standard riskless sense of the term: it requires taking a risk that the statistical regression model does not apply to the circumstances in the market *at the time* in which they plan to invest.

This argument leaves open the question of what are the exact model specifications that could replicate the empirically estimated mean forward discount bias parameter of −.88. We comment on this issue in Section 5 where we evaluate and qualify our results.

4.4 Sensitivity to the discount rate and to the coefficient of risk aversion

In this section we fix the endowment shocks in the foreign economy at the moderate levels of $\omega^{H*} = 31$ and $\omega^{L*} = 29$, and investigate the key volatility characteristics of interest in this paper when we vary the discount rate and the coefficient of risk aversion of the two agent-types. For each configuration we compute both the REE as specified in (4.1) as well as the RBE which we denoted in (4.3) as RBE (II). For each configuration of the parameters we report, in Tables 4 and 5 below, the respective equilibrium values of five central variables of interest which are:

(i) $\bar{r}^F$ – the riskless rate
(ii) ρ – the real equity premium
(iii) σ_{qd}^2 – the variance of the price/dividend ratio of the ownership shares
(iv) σ_s^2 – the variance of the foreign exchange rate
(v) β – the regression coefficient measuring the forward discount bias.

Starting with the effect of the discount rate, we report in Table 4 the values taken by the key variables in equilibria in which we fix $\gamma = \gamma^* = 3$ and vary the discount rates in the very wide range of .75–.95. The most noticeable conclusion that one can draw from Table 4 is that variations in the discount rate have very little effect on the equilibrium values taken by the key variables either in REE or in RBE. We have also calculated the cross effects of variations in the discount rate *together* with the coefficient of risk aversion but do not report these since they exhibit no significant interaction effects of such joint

Table 4. Comparison of volatility characteristics of RBE and REE for various discount rates

		$\delta^* = .75$		$\delta^* = .85$		$\delta^* = .95$	
		RBE	REE	RBE	REE	RBE	REE
$\delta = .75$	$\bar{r}^F$	1.81	4.88	1.81	4.84	1.80	4.81
	ρ	4.93	.60	4.84	.61	4.75	.61
	σ_{qd}^2	5.093	.436	4.963	.452	4.848	.466
	σ_s^2	.00951	.009E-5	.00909	.005E-5	.00878	.002E-5
	β	.245	.953	.244	.954	.244	.954
$\delta = .85$	$\bar{r}^F$	1.81	4.85	1.80	4.81	1.80	4.81
	ρ	4.84	.60	4.75	.60	4.67	.60
	σ_{qd}^2	4.965	.433	4.844	.449	4.735	.449
	σ_s^2	.00949	.013E-5	.00902	.008E-5	.00866	.005E-5
	β	.240	.954	.237	.954	.236	.954
$\delta = .95$	$\bar{r}^F$	1.81	4.81	1.80	4.78	1.79	4.74
	ρ	4.76	.60	4.67	.60	4.60	.60
	σ_{qd}^2	4.851	.430	4.737	.446	4.634	.460
	σ_s^2	.00951	.018E-5	.00901	.012E-5	.00862	.008E-5
	β	.237	.954	.233	.954	.231	.955

Table 5. Comparison of volatility characteristics of RBE and REE for various coefficients of risk aversion

		$\gamma^* = 2.5$		$\gamma^* = 5$	
		RBE	REE	RBE	REE
$\gamma = 2.5$	$\bar{r}^F$	2.11	4.89	1.68	4.69
	ρ	4.03	.48	4.92	.68
	σ^2_{qd}	3.282	.419	5.372	.477
	σ^2_d	.00610	.006E-5	.02232	.224E-5
	β	.192	.953	.383	.955
$\gamma = 5.0$	$\bar{r}^F$	1.66	4.69	.65	4.30
	ρ	4.93	.67	6.90	1.07
	σ^2_{qd}	5.318	.471	9.702	.541
	σ^2_s	.01765	3.307E-5	.02452	.013E-5
	β	.383	.955	.405	.959

variations. These results are similar to those reported by Mehra and Prescott [1985].

Turning to the coefficient of risk aversion, we consider in Table 5 the following parameter configurations for the two agent-types: fix $\delta = \delta^* = .9$ and consider the combinations

$$\gamma = 2.5 \qquad \gamma^* = 2.5$$
$$\gamma = 2.5 \qquad \gamma^* = 5.0$$
$$\gamma = 5.0 \qquad \gamma^* = 2.5$$
$$\gamma = 5.0 \qquad \gamma^* = 5.0.$$

One notes that the difference between $\gamma = 2.5$ and $\gamma = 5$ is significant enough to be of interest but cannot be viewed as "drastic".

Table 5 shows that the various measures of volatility exhibit some interesting changes across the four different REE's and RBE's but these changes are not drastic in nature.

Summary. We can summarize the five central results of this paper regarding the volatility of foreign exchange rates:

1. The volatility of foreign exchange rates originates in the motive for optimal porfolio adjustment and thus mostly reflects the movement of financial assets.
2. Endogenous uncertainty is a major explanation of the volatility of foreign exchange rates. This uncertainty is generated neither by the exogenous variables nor by monetary shocks in the two economies: it is propagated by

the expectations of the agents and impacts the markets *in addition* to the volatility induced by monetary policy.

3. The RBE theory offers natural explanations for all three empirical phenomena discussed in this paper.

4. The essential conditions which we needed to generate the three observed phenomena are price effects on the probabilities at each state and private signals which are correlated.

5. Trade in the world economy involves multiple commodities: many commodities which are imported by a country are not exported by the same country and a large number of commodities are not traded internationally at all. Hence, variations of the exchange rate must have a limited effect on the domestic price level even if all prices are completely flexible. However, the mechanism of endogenous propagation of volatility of the foreign exchange rate as developed in this paper remains intact and this shows that the Dornbusch [1976] assumption of slow domestic price adjustment is not needed for an explanation of the volatility of the foreign exchange rate.

5 Some additional discussion and qualifications

5.1 Some comments on the Dornbusch [1976] Theory

The Dornbusch [1976] model has been most influential in providing a persuasive argument that in an REE one must think of fluctuations of the foreign exchange rate as an overshooting phenomenon in response to monetary shocks. Indeed, our results in Table 1 show clearly that in an REE with price flexibility one cannot expect to have fluctuations of the exchange rate. It is then clear that the REE based theory offered by Dornbusch [1976] for the volatility of exchange rates is drastically different from the theory offered in this paper. In contrast to a theory that views the volatility of exchange rates as overshooting reactions to monetary shocks, our theory considers the volatility of the exchange to be endogenously propagated and contends that *exchange rate volatility would arise even if no shocks to monetary policy ever occurred.*

Does the evidence support the Dornbusch [1976] theory? Without a major survey of the empirical evidence we note first that uncovered interest parity is an essential component of the expectational assumptions of the Dornbusch [1976] theory and the forward discount bias is the major *indirect* evidence against this theory. One may attempt to bolster the Dornbusch [1976] theory by suggesting that slow price response and hence overshooting of the foreign exchange rate results from learning by agents. The learning literature is very extensive and we cannot survey it here. Instead, we mention the example of a learning process in the foreign exchange markets formulated by Lewis [1989]. This paper attempts to explain the systematic under-prediction of the strength of the dollar during the period 1980–1985. Lewis [1989] assumes that agents know both the true equilibrium process of the exchange rate as well as

the true probability distribution of the fundamental variables which enter the exchange rate determination mechanism. The problem is then reduced to estimating from the data a single parameter which represents the shift of the mean value of the equilibrium process. According to the REE based exchange rate determination model, knowledge of the correct parameter by market participants would have lead to the correct prediction of the strength of the dollar during the years under study.

Lewis [1989] makes very strong assumptions in specifying the structural knowledge of market participants and the true equilibrium map which determines the exchange rate. Nevertheless, her model cannot correct for more than half of the *presumed* bias (under the REE based pricing) in the market forecast of the exchange rate. More fundamental is the observation that under the assumptions made by Lewis [1989] and other learning models, the forward discount bias *should always show a tendency to converge to the REE level* after any change in policy regime. This is contradicted by the evidence since in the real data of the economy, the forward discount bias persists exactly as it does in the RBEs of Tables 2–5. In fact, Lewis [1989] treats every new policy regime as if it were a once-and-for-all policy change which has nothing to do with any previous change in policy regimes. Thus, the prior distribution of the unknown parameter used by the agent does not benefit from any knowledge that he may gain from the extensive data of the past. Suppose the agent takes care to infer from the policy parameters experienced in the past to determine what would be the best prior to use. Then, given the structural knowledge at his disposal, his posterior could converge much faster than exhibited in the calculations of Lewis [1989]. The idea of using a "rational" prior is not in the Bayesian spirit. It is, however, exactly the first step needed for the learning agent in Lewis [1989] to hold a rational belief. We thus conclude that the learning program outlined by Lewis [1989] is deficient on many levels and would fail under the non-stationarity conditions postulated in our theory. This is confirmed by the observed persistence of exchange rate volatility and forward discount bias which cannot be explained by monetary shocks.

The recent paper by Eichenbaum and Evans [1995] provides *direct* empirical evidence against the overshooting theory of exchange rate volatility. It shows that contrary to the Dornbusch [1976] theory, a contractionary U.S. monetary policy leads to significant, persistent appreciation in U.S. nominal and real exchange rates and a significant, persistent forward discount bias. These empirical findings are interesting in *suggesting to us* that an important component of exchange rate determination are the expectations of the agents about *future U.S. monetary policy*. Moreover, agents may have diverse interpretations of the significance of any shift in U.S. monetary policy at date t to their predictions of the policies that will be adopted in the future. Such effects of present policies on the *expectations* of future policies are absent from the REE theory of Dornbusch [1976] but can be incorporated into an expanded version of our model in a natural way.

5.2 Some qualifications .

General equilibrium analysis with nominal assets raises several complex problems which we addressed only partly here and for this reason the present paper may be viewed only as a first stage exploration of the volatility characteristics of foreign exchange rates. In this brief section we discuss two questions which need some additional clarification.

a. Insufficient nominal assets

The most significant weakness of the model presented above is the insufficiency of nominal assets. The clearest consequence of this difficulty is the fact that our model does not have nominal equilibrium interest rates for the two economies. For this reason we approximated the key forward discount bias parameter β by using the expressions (28a)–(28b) instead of the correct, equilibrium, nominal interest rates which we did not have. In addition, we stress the fact that our model did not yield results of the bias parameter which are close to the average estimated forward discount bias parameter of $-.88$ reported by Froot [1990]. We believe that this, more demanding task, should be attempted only with a model containing riskless nominal debt instruments in both economies. In such a model one would examine the structure of beliefs that could give rise to negative values of the bias parameter.

Expanding the model by adding riskless nominal debt instruments is, however, no minor task. An earlier formulation of our model called for a more complex financial structure consisting of two nominal bonds, two distinct stocks and one real bond. Such models are computationally difficult to solve because of their complex financial structure and implied size. To see why, note that the model used in this paper is described by a system of 128 equations. The addition of one financial asset to the model requires the addition of 48 equations: 16 first order conditions for each agent-type and 16 market clearing equations for the 16 different states. Hence, the addition of two nominal bonds would have required us to solve equilibria with 224 equations. This added financial complexity combined with the increased size of the model made the solution infeasible for us when "feasibility" means carrying out a research agenda according to which one needs to compute different equilibria for many different configurations of the parameters. However, this situation is rapidly changing with the development of more powerful methods of solving equilibria of models containing complex financial assets and in which significant problems of arbitrage arise. We thus expect that in the near future models of RBE with a much richer financial structure could be analyzed numerically.

The lack of adequate financial assets may also be related to the fact that the RBE's reported in the simulations above tended to exhibit levels of price volatility which we considered as too high relative to the observed price level volatility in the major economies of the world. Keep in mind that in the model used in this paper the two assets are real since they both pay in real terms. Consequently, an interesting question arises in the context of an RBE: suppose we introduce *nominal* assets into the model and thus give the agents the

opportunity to trade their perceived uncertainty of price level risk. What effect would this change have on the equilibrium level of volatility of the price levels p_t and p_t^* in the two economies? This question is related to the more general problem of assessing the effect of a change in market structure on the volatility characteristics of any equilibrium under consideration. It is clear that in an economy with an incomplete market structure (in the standard sense of an Arrow-Radner economy) the introduction of a new financial asset will have some effect on the level of volatility of an REE. However, in such an REE all market volatility is still perfectly correlated with exogenous variables; an incomplete market structure does not alter the exogenous uncertainty characteristic of an REE. In the context of an RBE there are two points to be noted. First, as was shown by Kurz and Wu [1996], endogenous uncertainty is generically present in equilibrium even if the market structure of an RBE is complete. Under these conditions the introduction of an *additional* financial asset will have no real effect as long as the structure of beliefs does not change. This result is the same as in the case of an REE. If the market structure in an RBE is not complete then the addition of a financial asset will have a real effect on consumption and on portfolio choices which depends upon the structure of beliefs of the agents. Moreover, our second observation is that RBE are generically incomplete since for any set of financial assets we can always find a structure of private signals and states of belief which will render the given financial structure incomplete. This naturally complicates the study of the effect of changes of financial structure on the volatility characteristic of an RBE.

b. Allowing for imperfect correlation within the same agent-type

The agent-type in the model above was interpreted to represent a large number of identical agents of the same type. However, with only two states of belief for each agent-type, this means that all the agents of the same type have *perfectly* correlated private signals and this is not reasonable. Here again, the issue is not one of theoretical tractability but rather computational feasibility. For example, consider an RBE without perfect correlation among the private signals of all members of an agent-type with two dividend states, two states of the foreign exogenous shocks and three states of the private signals of each agent-type. Such an RBE has 36 prices and is represented by a system of 288 equations (which can be analytically reduced to 216 equations).

Leaving the computational feasibility aside for a moment, we want to conclude by stressing that increasing the social states of each agent-type is a very important step. It moves us closer to the fundamental observation which we have made on several occasions: contrary to an initial impression that the beliefs of agents matter, it appears that the belief of *any one agent* has little significance to market behavior. What really matters is the distribution of probability beliefs among members of each agent-type and across types. More specifically, what matters is the joint distribution of social states of private signals where "social" is broken into two components. The first component is

the joint distribution of beliefs *among members of each agent-type* since this determines the structure and the number of aggregate states of each type. The second component is the joint distribution of aggregate states across types. This joint distribution determines the structure of aggregate states of belief in the economy and their effect on the volatility characteristics of any RBE.

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